Reflection and Transmission at Oblique Incidence

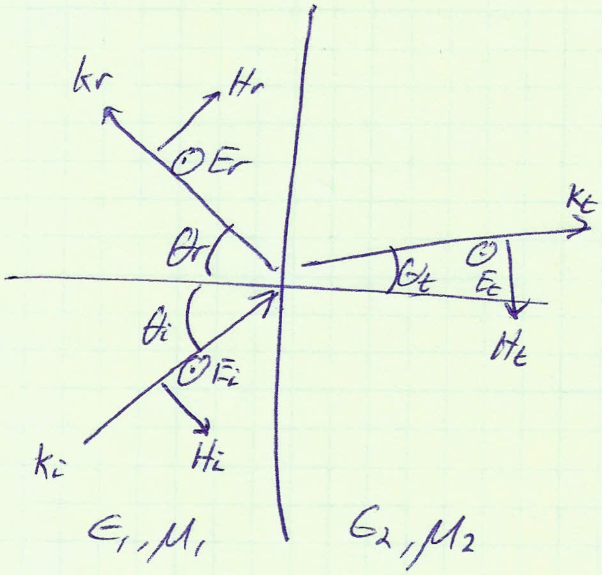
Terminology:

plane of incidence: plane containing normal to boundary and direction of propagation of incident wave

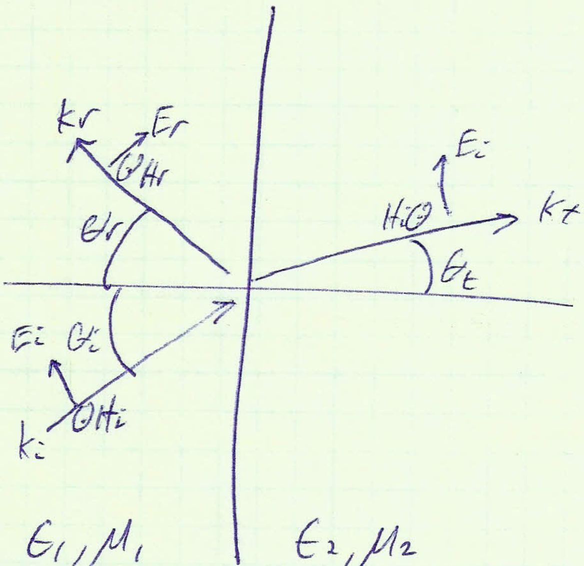
parallel polarization: E-field is parallel to plane of incidence (also called TM)

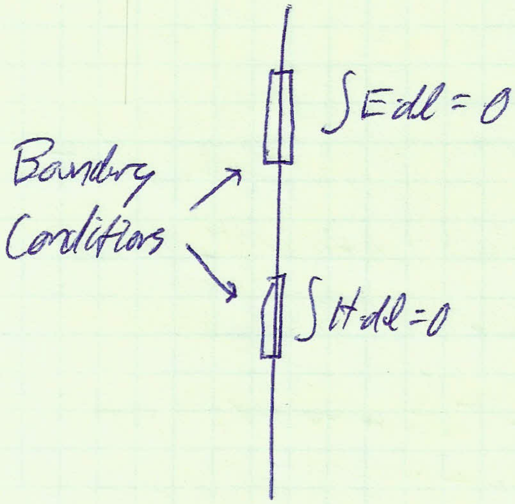
perpendicular polarization: E-field is perpendicular to plane of incidence (also called TE)

perpendicular (TE)

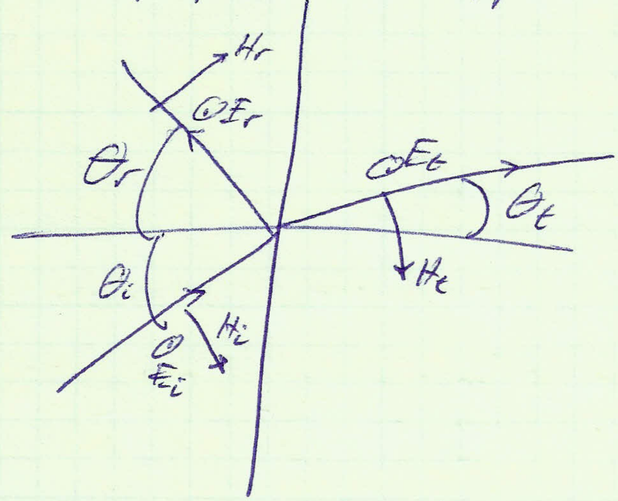


parallel (TM)





for perpendicular (TE) polarization:



Ignore the complicated change of coordinate system in the book
Just look at the tangential component of E and H at the surface

$$E_i + E_r - E_t = 0 \longrightarrow E_t = E_i + E_r$$

$$H_i \cos\theta_i - H_r \cos\theta_r - H_t \cos\theta_t = 0$$

$$\frac{E_i}{\eta_1}\cos\theta_i - \frac{E_r}{\eta_1}\cos\theta_r - \frac{E_t}{\eta_2}\cos\theta_t = 0$$

$$\frac{E_i}{\eta_1}\cos\theta_i - \frac{E_r}{\eta_2}\cos\theta_r - \frac{E_i + E_r}{\eta_2}\cos\theta_t = 0$$

$$E_i\left(\frac{\cos\theta_i}{\eta_1} - \frac{\cos\theta_t}{\eta_2}\right) - E_r\left(\frac{\cos\theta_r}{\eta_i} + \frac{\cos\theta_t}{\eta_2}\right) = 0$$

$$\Gamma_\perp = \frac{E_r}{E_i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_r + \eta_1\cos\theta_t}$$

similar procedure can find

$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}, \quad \tau_\perp = 1 + \Gamma_\perp$$

note – you might think $\Gamma + \tau = 1$ – wrong – remember Γ can have either sign

also, don't worry about $1 + \Gamma > 1$ → power scales with $\frac{1}{\eta}$

Still use Snell's laws: $\theta_r = \theta_i$, $\dfrac{\sin\theta_t}{\sin\theta_i} = \dfrac{n_1}{n_2}$

using this

for <u>non</u>magnetic dielectrics:

$$\Gamma_\perp = \frac{\cos\theta_i - \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}$$

- Parallel (TM) polarization:

same procedure - match transverse components of E and H

$$\Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\tau_\parallel = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \rightarrow \tau_\parallel = (1 + \Gamma_\parallel)\frac{\cos\theta_i}{\cos\theta_t}$$

for <u>non</u>magnetic dielectrics:

$$\Gamma_\parallel = \frac{-(\epsilon_2/\epsilon_1)\cos\theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}{(\epsilon_2/\epsilon_1)\cos\theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}$$

Brewster angle $\rightarrow \Gamma = 0$

for perpendicular: $\eta_2 \cos\theta_i = \eta_1 \cos\theta_t$

using Snell's law $\rightarrow$ $\sin\theta_{B\perp} = \sqrt{\dfrac{1 - (\mu_1\epsilon_2/\mu_2\epsilon_1)}{1 - (\mu_1/\mu_2)^2}}$

for nonmagnetic materials, $\mu_1 = \mu_2$

$\rightarrow$ Brewsters angle does not exist

Brewsters angle for parallel polarization

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_i$$

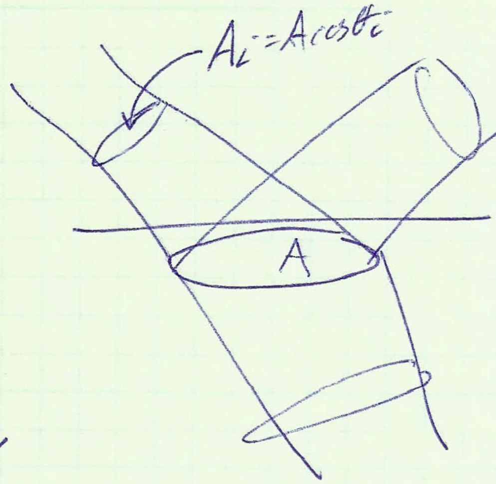
using snell's law,

$$\sin\theta_{B\parallel} = \sqrt{\frac{1-(\epsilon_1\mu_2/\epsilon_2\mu_1)}{1-(\epsilon_1/\epsilon_2)^2}}$$

for nonmagnetic materials,

$$\sin\theta_{B\parallel} = \sqrt{\frac{1}{1+(\epsilon_1/\epsilon_2)}} \rightarrow \theta_{B\parallel} = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

- Reflectivity (reflectance) and Transmissivity (transmittance)



power density $S = \frac{|E_0|^2}{2\eta}$

Area of each beam $A_i = A\cos\theta_i$
same for reflected, transmitted

$$P_i = S_i A_i = \frac{|E_0^i|^2}{2\eta_1} A\cos\theta_i$$

same for reflected, transmitted

$$R_\perp = \frac{P_\perp^r}{P_\perp^i} = \frac{|E_{\perp 0}^r|^2\cos\theta_r}{|E_{\perp 0}^i|^2\cos\theta_i} = \left|\frac{E_{\perp 0}^r}{E_{\perp 0}^i}\right|^2 \rightarrow R_\perp = |\Gamma_\perp|^2$$

similarly, $R_\parallel = |\Gamma_\parallel|^2$

Transmittance:

$$T_\perp = \frac{P_\perp^t}{P_\perp^i} = \frac{|E_{\perp 0}^t|^2\eta_1}{|E_{\perp 0}^i|\eta_2}\frac{A\cos\theta_t}{A\cos\theta_i} = |\tau_\perp|^2\left(\frac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i}\right)$$

- Conservation of Power:

$$R_\perp + T_\perp = 1 \qquad R_\parallel + T_\parallel = 1$$